Heat pipe problem

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1 Introduction

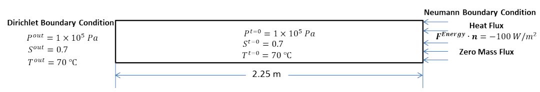
When an unsaturated porous medium is subjected to a constant heat flux and the temperature is sufficiently high, water is heated and vaporizes. Vapor flows under its pressure gradient towards the cooler end where it condenses. Vaporization and condensation produce a liquid saturation gradient, creating a capillary pressure gradient inside the porous medium. Condensate flows towards the hot end under the influence of a capillary pressure gradient. This is a heat pipe in an unsaturated porous medium.

A benchmark regarding the heat pipe problem was proposed by Udell and Fitch (1985). A semi-analytical solution was provided for a non-isothermal water–gas system in a porous medium, in which the heat convection, conduction, and diffusion as well as capillary effects play a key role.

1.1 Physical scenario

As shown in Figure 1, the heat pipe was represented by a 2D horizontally column (2.25m in length and 0.2m in diameter) of porous media, which was partially saturated with a liquid phase saturation value of 0.7 at the beginning. a heater is installed on the right-hand-side of the horizontal column with an initial water saturation of 0.5. It generates a constant heat flux of $100\,\mathrm{W\,m^{-2}}$ and raises the temperature gradually above the boiling point. At the left-hand boundary, we impose the constant gas phase pressure (pg = 101330 Pa) as Dirichlet boundary condition. Here, both thermal convection and conduction are considered along with the latent heat transfer, i.e. evaporation and condensation.

Figure 1: Geometry of the Heat Pipe Problem.



1.2 Model parameters and numerical settings

In this benchmark, the heat conductivity for an unsaturated medium is given as

$$\lambda(S_G) = \lambda_{\mathrm{pm}}^{S_L=0} + \sqrt{(1-S_G)}(\lambda_{\mathrm{pm}}^{S_L=1} - \lambda_{\mathrm{pm}}^{S_L=0}). \tag{1}$$

The Leverett function (Leverett et al. (1941)) and Brooks-Corey relationship (Brooks and Corey (1964)) are applied to describe the dependency of capillary pressure and relative permeability on saturation. While for the fluid properties of water, the IAPWS (Wagner and Pruß (2002)) formulation is applied. In this context, the parameters used in this benchmark are listed in Table 1.

Table 1: Parameters applied in the Heat Pipe problem

Parameters Name	Symbol	Value	Unit
Intrinsic Permeability	K	10^{-12}	m^2
Porosity	ϕ	0.4	–
Latent heat of vaporization of water	$h_{\Delta e}$	2258	$kJ\ kg^{-1}$
Heat conductivity of fully saturated porous medium	$\lambda_{\mathrm{pm}}^{S_L=1}$	1.13	$W\ (m\ K)^{-1}$
Heat conductivity of dry porous medium	$\lambda_{\mathrm{pm}}^{S_L=0}$	0.582	$W\ (m\ K)^{-1}$
Heat capacity of the soil grains	c_{s}	700	$J\ (kg\ K)^{-1}$
Density of the soil grain	ρ^{s}	2600	$kg\ m^{-3}$
Density of the water	ρ^{w}	1000	$kg\ m^{-3}$
Density of the air	ρ^{a}	0.08	$kg\ m^{-3}$
Dynamic viscosity of water	μ^{w}	$2.938 \cdot 10^{-4}$	$Pa \cdot s$
Dynamic viscosity of air	$\mu_{\mathrm{G}}^{\mathrm{a}}$	$2.08 \cdot 10^{-5}$	$Pa \cdot s$
Dynamic viscosity of steam	$\mu_{\mathrm{G}}^{\mathrm{w}}$	$1.20 \cdot 10^{-5}$	$Pa \cdot s$
Diffusion coefficient of air in gas	$D_{\mathrm{G}}^{\mathrm{a}}$	$2.6 \cdot 10^{-5}$	$m^2\ s^{-1}$
Diffusion coefficient of air in liquid water	$D_{\mathrm{L}}^{\mathrm{a}}$	$3 \cdot 10^{-9}$	$m^2\ s^{-1}$

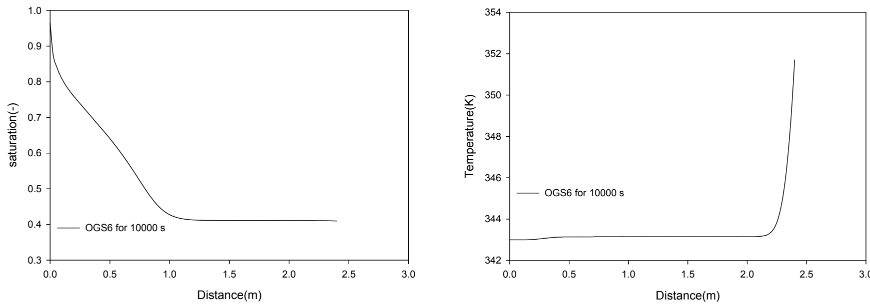
1.3 Results and analysis

In the CTEST-small, the comparison is made for the time of 10000 seconds. The profiles of saturation and temperature are plotted as Figure.(2) shown.

In the CTEST-large, the comparison is made for the time of 1.1e+6 seconds. Around this time, the water is fully evaporated from the heating boundary(right hand side), and one phase zone of gas phase is formulated. While the temperature at this part begin to increase significantly. These can be observed from Figure.(3).

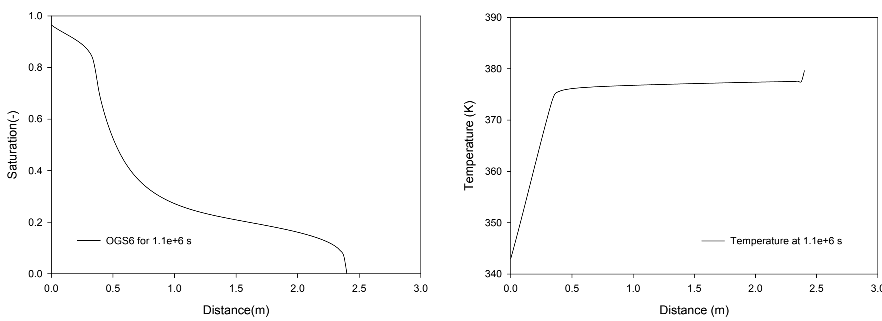
After the gas phase appearance, the time step size decrease dramatically in order to assure the numerical stability. In this test, 10 seconds is applied.

We further compare the simulation results against the semi- analytical solution in Figure.(4) for the steady state. A good agreement can be observed with respect to temperature and saturation. However, still some discrepancy can be found for the saturation profile especially at the "cold" region(the left hand side boundary).



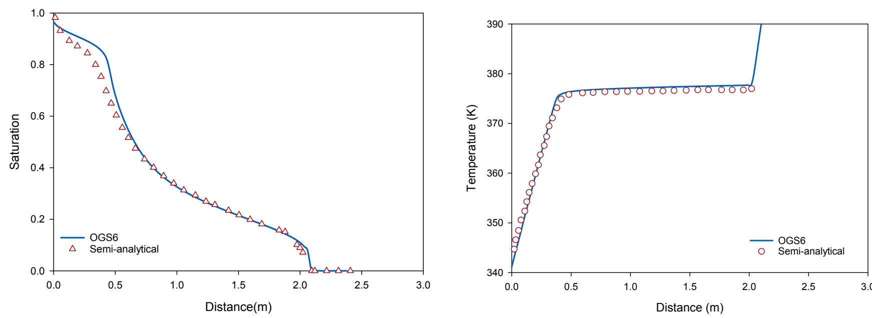
(a) (b)

Figure 2: Profile for saturation and temperature at 10000 seconds over the simulation domain



(a) (b)

Figure 3: Profile for saturation and temperature at 1.1e+6 seconds over the simulation domain



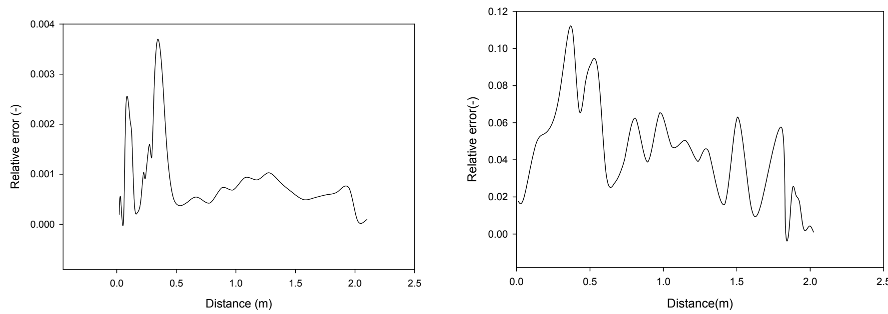
(a) (b)

Figure 4: Profile of saturation and temperature over simulation domain compared against semi-analytical solution.

Those might be owing to the following facts. In the original semi-analytical solution configuration, the boundary conditions for the left hand side are set for both saturation and molar fraction of gas air together(besides gas pressure and temperature), while in OGS6 solution, the boundary conditions are only imposed for capillary pressure(besides gas pressure and temperature) since PP-scheme is applied, the saturation and molar fraction are thereby treated as secondary variables.

With the given initial conditions, the model could reproduce the heating at the right-hand boundary from 70 °C up to boiling temperature. Also, the gradual extension of the heat pipe region until the stationary system state was reached was modeled correctly. The disappearance of the water phase associated with a change of the phase state was carried out well. The OGS6 solution allows a region near the heated boundary to completely dry out, thus creating increasing temperatures, in comparison to the semi-analytical results.

Next we further plot relative error of simulation results against semi-analytical solution with respect to temperature and saturation, respectively(see Figure.5).



(a) relative error of temperature

(b) relative error of saturation

Figure 5: Relative error comparison between simulation results against semi-analytical solution at steady status.

References

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