Hydromechanics with a fracture

This document describes a hydromechanics problem in porous media having a pre-existing fracture.

1. Governing equations

Consider porous media consisting of solid and liquid phases. Under assumptions of small deformation and the laminar fluid flow, hydromechanical processes in the porous media can be formulated as

$$\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g} = 0 \quad \text{on } \Omega \tag{1.13}$$

$$S_s \frac{\partial p}{\partial t} + \alpha \mathbf{m}^T \cdot \frac{\partial \boldsymbol{\epsilon}}{\partial t} + \nabla \cdot \mathbf{q} = 0 \quad \text{on } \Omega \tag{1.14}$$

$$\boldsymbol{\sigma} = \boldsymbol{\sigma'} - \alpha \mathbf{m}\, p \tag{1.15}$$

$$\boldsymbol{\sigma'} = \mathbf{D}\, \boldsymbol{\epsilon} \tag{1.16}$$

$$\boldsymbol{\epsilon} = \frac{1}{2}\left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right) \tag{1.17}$$

$$\mathbf{q} = -\frac{\mathbf{k}}{\mu}(\nabla p + \rho^l \mathbf{g}) \tag{1.18}$$

with boundary conditions

$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on } \Gamma_{D_u}, \tag{1.19}$$

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \bar{\mathbf{t}} \quad \text{on } \Gamma_{N_u} \tag{1.20}$$

$$p = \bar{p} \quad \text{on } \Gamma_{D_p}, \tag{1.21}$$

$$\boldsymbol{q} \cdot \mathbf{n} = \bar{\mathbf{q}} \quad \text{on } \Gamma_{N_p} \tag{1.22}$$

where $\boldsymbol{\sigma}$ is the stress tensor, ρ is the effective density of porous media, $\mathbf{g}$ is the gravitational vector, S_s is the specific storage, p is the pore pressure, α is the Biot coefficient, $\mathbf{m}$ is the identity vector for mapping non-shear components, $\boldsymbol{\epsilon}$ is the strain vector, $\mathbf{q}$ is the Darcy flux vector, $\boldsymbol{\sigma'}$ is the effective stress tensor, $\mathbf{D}$ is the fourth order elastic material tensor, $\mathbf{u}$ is the solid displacement vector, $\mathbf{k}$ is the permeability tensor, μ is the fluid dynamic viscosity, $\bar{\mathbf{u}}$ is the prescribed displacement, $\bar{\mathbf{t}}$ is the traction force vector, $\bar{\mathbf{p}}$ is the prescribed pore pressure, and $\bar{\mathbf{q}}$ is the incoming/outgoing Darcy velocity.

If a fracture exists on Γ_d under compressive stress, the stress equilibrium equation has to satisfy

$$\mathbf{t}_d^+ = -\mathbf{t}_d^- = (\boldsymbol{K}\boldsymbol{w} - \alpha \mathbf{m}^T p) \quad \text{on } \Gamma_d \tag{1.23}$$

where $\mathbf{t}_d^+, \mathbf{t}_d^-$ are traction forces acting on the fracture upper and lower surfaces, $\mathbf{K}$ is the fracture constitutive matrix, $\boldsymbol{w}$ is the fracture relative displacement in the local coordinates along the fracture plane, which is given as

$$\boldsymbol{w} = \boldsymbol{R}_d[[\boldsymbol{u}]] \tag{1.24}$$

with the rotation matrix $\mathbf{R}_d$ and the relative displacement in global coordinates

$$[[\boldsymbol{u}]] = \boldsymbol{u}^+ - \boldsymbol{u}^- \quad \text{on } \Gamma_d. \tag{1.25}$$

The fracture relative displacement changes the fracture aperture as

$$b = b_0 + w_n \geq 0 \tag{1.26}$$

with the initial mean aperture b_0. For the liquid flow process, assuming the fluid mainly flows along the fracture, the following additional equation has to be considered as

$$bS_s \frac{\partial p}{\partial t} + \alpha \frac{\partial b}{\partial t} + \nabla \cdot \left(-\frac{b^3}{12\mu} (\nabla p + \rho^l \mathbf{g}) \right) = 0 \quad \text{on } \Gamma_d \tag{1.27}$$

with a continuity condition across the fracture

$$p^+ = p^- \quad \text{on } \Gamma_d. \tag{1.28}$$

It should be noted that the continuity condition is inappropriate if the fracture has some low permeable infills.

2. Example

This example considers fluid injection into a deformable fracture surrounded by an impermeable rock matrix. As illustrated in Figure 1.1, the major fracture lies horizontally in the middle of an impermeable rock block. The fracture is subjected to a uniform in-situ stress σ_{yy} normal to the fracture. Initially, fracture aperture is uniformly $b_0 = 1.0 \times 10^{-2}$ mm and fluid pressure is $p_0 = 11.0$ MPa along the fracture. At time $t = 0^+$, fluid is injected at the left-most edge of the fracture (in the form of constant boundary pressure, $p = 11.9$ MPa) and a sudden increase of pressure in the fracture results. The injection pressure induces elastic fracture opening and a subsequent increase of fracture permeability and storage capacity. Material parameters are listed in Table 1.1. Stress in the surrounding rocks is 50 MPa and fluid pressure is 11 MPa. Boundary fluid pressure is fixed at $t = 0^+$ to 11.9 MPa at the left and 11 MPa at the right.

Simulation results are presented in Figure 1.2 for pressure and fracture aperture profile along the fracture. Once the fluid is injected, the fracture aperture is instantaneously opened to nearly 1.9 $\times 10^{-2}$ mm at the injection point ($x = 0$ m). With time, this fracture opening behavior gradually propagates toward the right-most, low-pressure edge of the fracture. Linear constitutive laws dictate a linear variation in fracture aperture relative to fluid pressure.

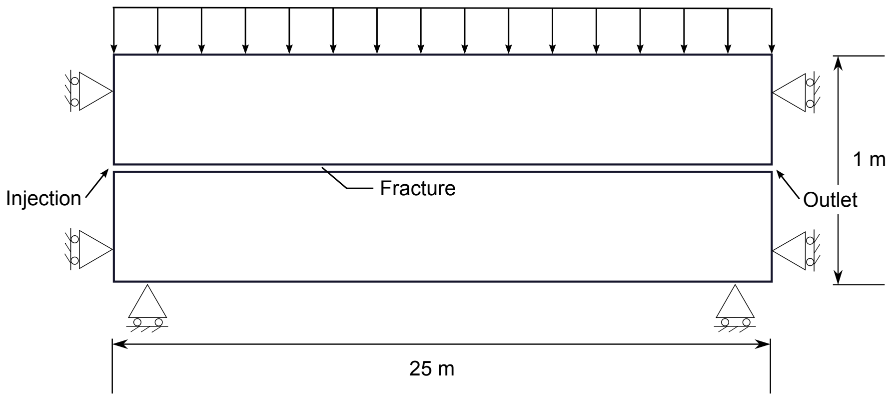

Figure 1.1: Problem definition

Table 1.1: Example 3: material parameters

Fluid	
Density	1000.0 kg/m^3
Viscosity	0.001 Pa s
Rock (porous medium)	
Density	2716.0 kg/m^3
Specific storage	1.0×10^{-10} Pa^{-1}
Permeability	1.0×10^{-21} m^2/s
Porosity	0.1 %
Young's modulus	60 GPa
Poisson ratio	0.0
Biot constant	1.0
Fracture	
Initial aperture	1.0×10^{-5} m
Specific storage	0.0 Pa^{-1}
Joint normal stiffness	100 GPa/m
Joint shear stiffness	100 GPa/m
Biot constant	1.0

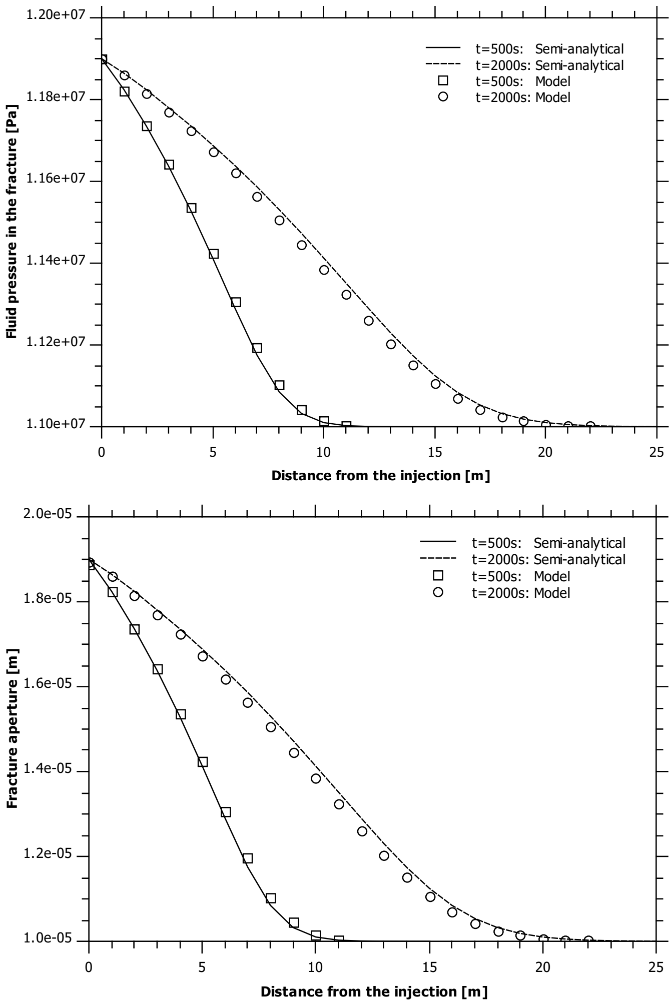

Figure 1.2: Results: profiles along the fracture, pressure (left) and aperture (right)